Department of Computer Science
CMPT 250.6
Midterm Exam
CLOSED BOOK

Time: 50 minutes

March 13, 2002

Marks

(10) 1. Use mathematical induction to prove that $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1$ whenever n is a nonnegative integer.

(8) 2. In class, we discussed the open addressing method of resolving collisions. In particular, we covered linear probing, random probing and double hashing techniques. Another probing method is called quadratic probing. Suppose that a key value initially hashes to position d and a collision results. On its first attempt to resolve the collision, the quadratic algorithm attempts to place the key at position:

$d + 1^2$

If a second attempt is necessary to resolve the collision, position:

$d + 2^2$

is probed. In general the rth attempt to resolve the collision probes position

$d + r^2$

with wraparound taken into account.

Using the hashing function

$H(x) = (x \bmod 10) + 1$

with a table size of 10 and an initial hash position of 6, which locations will never be probed when a collision occurs.

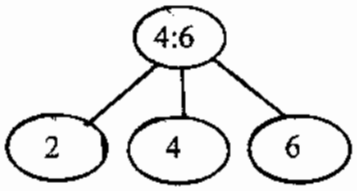
(10) 3. Give the Eiffel code to define a routine called *print_range* which will print the keys in an ordered binary tree in the range from values given by the arguments *low* to *high* inclusive.

This routine is to be written for inclusion as a feature of LINKED_SIMPLE_TREE_UOS[G].

Recall that the main features of LINKED_SIMPLE_TREE_UOS[G] are:
Note that to shorten the description, LS_TREE[G] is used instead of LINKED_SIMPLE_TREE_UOS[G].

is_empty : BOOLEAN
is_full : BOOLEAN
out : STRING
make
initialize (lt:LS_TREE[G]; x:G; rt:LS_TREE[G])
root_left_subtree : LS_TREE[G]
root_right_subtree : LS_TREE[G]
root_item : G

(8) 4. Given the 2-3 tree



Draw the tree after each of the following operations. If rebalancing was necessary, draw the tree at each stage of the rebalancing process.

Insert 3
Delete ~~Insert~~ 6
Insert 8
Insert 9
Insert 7
Delete ~~Insert~~ 2

(6) 5. Suppose that you are to design test cases for a *deletion* procedure that is in a class for a height-balanced binary tree. Describe each of the situations that you should test *which pertains to* a height-balanced binary tree.

NOTE: There are many tests cases that you did in your assignment that pertains to an ordered binary tree. THESE ARE NOT TO BE REPEATED! ONLY THE EXTRA ONES THAT PERTAIN TO A HBBT NEED BE GIVEN!

(8) 6. Formulate a context-free grammar that generates the set of nonnegative *even* integers. This language consists of nonnegative integers that must end with 0, 2, 4, 6, or 8.